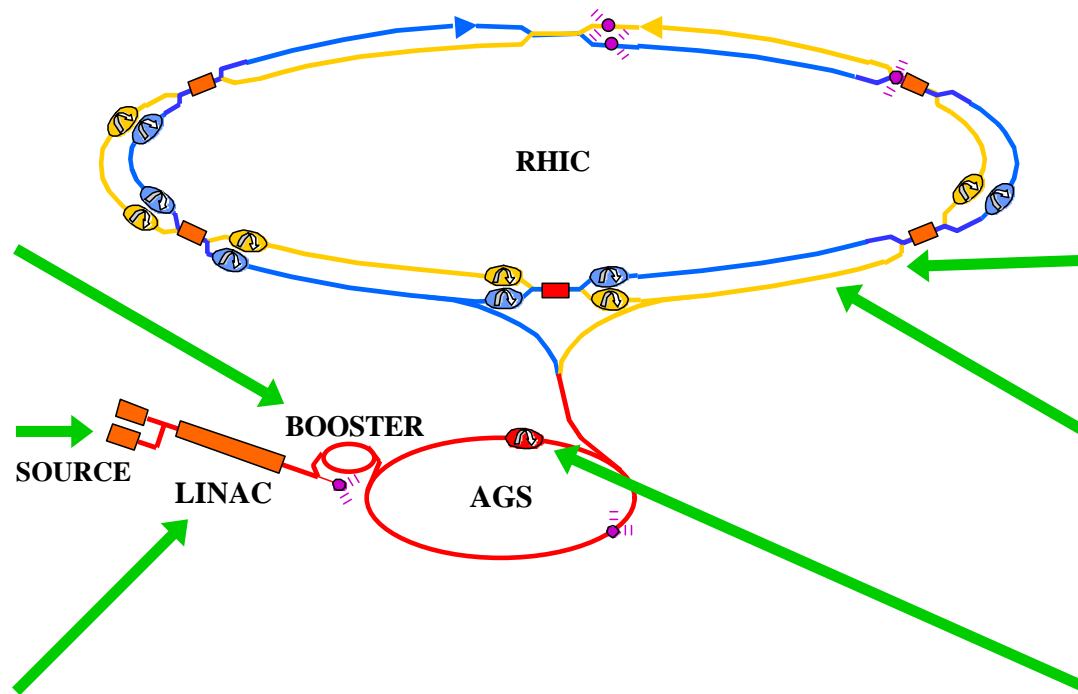
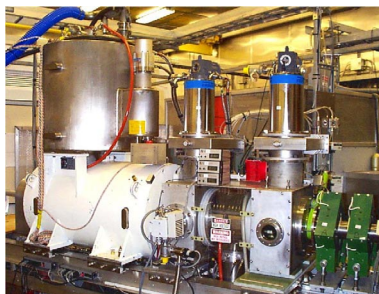


eRHIC ZDR: Polarized Ions

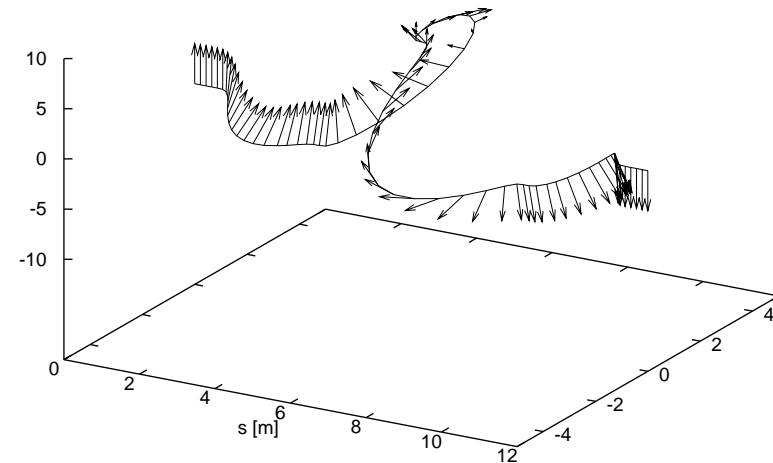
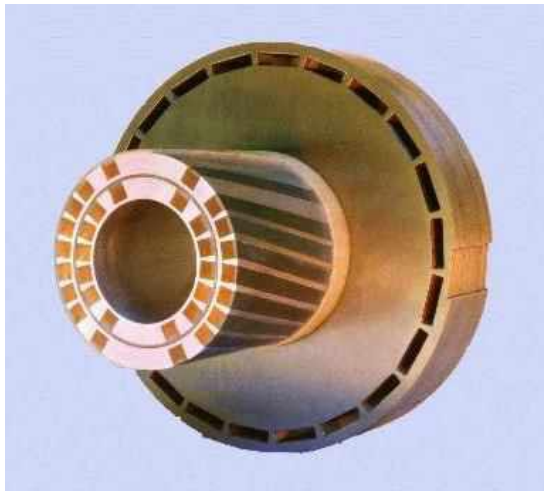
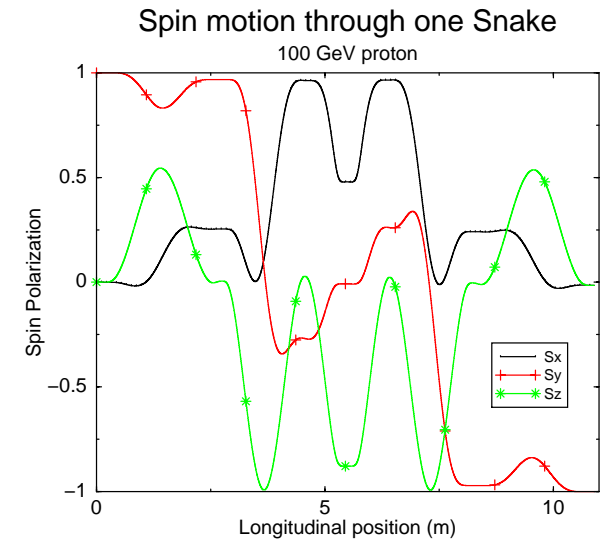
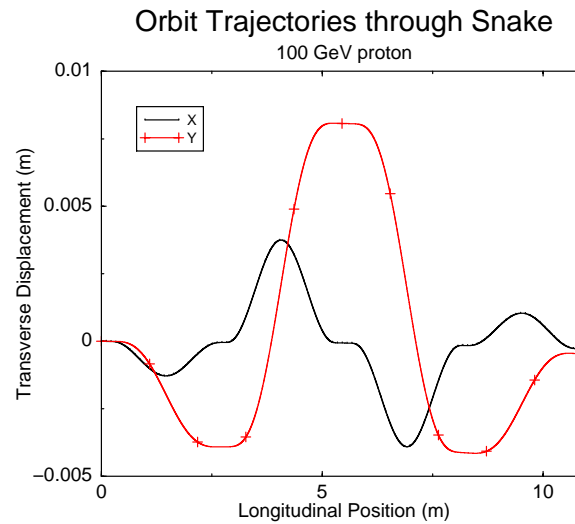
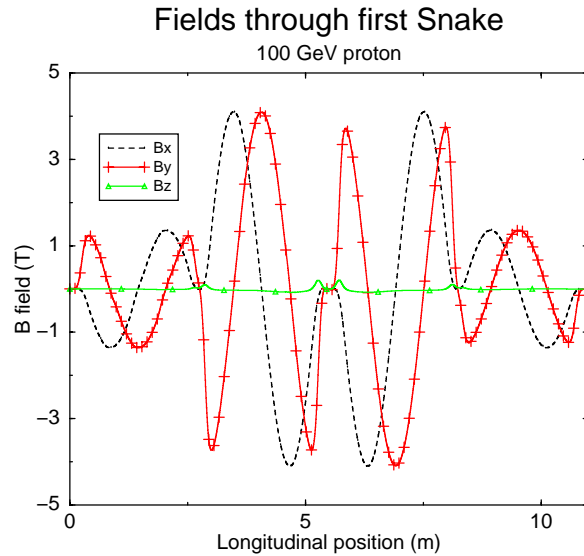
Waldo MacKay and Mei Bai

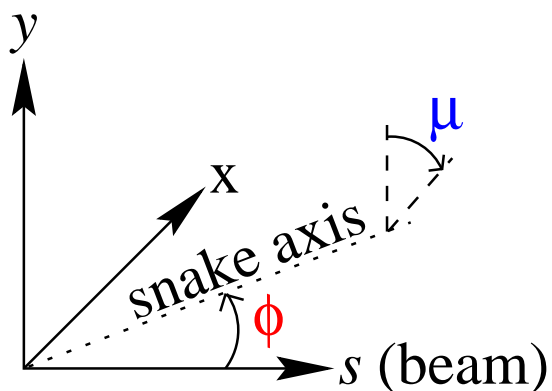
Accelerator Complex (Pol. Protons)



LINAC: Linear Accelerator
AGS: Alternating Gradient Synchrotron
RHIC: Relativistic Heavy Ion Collider

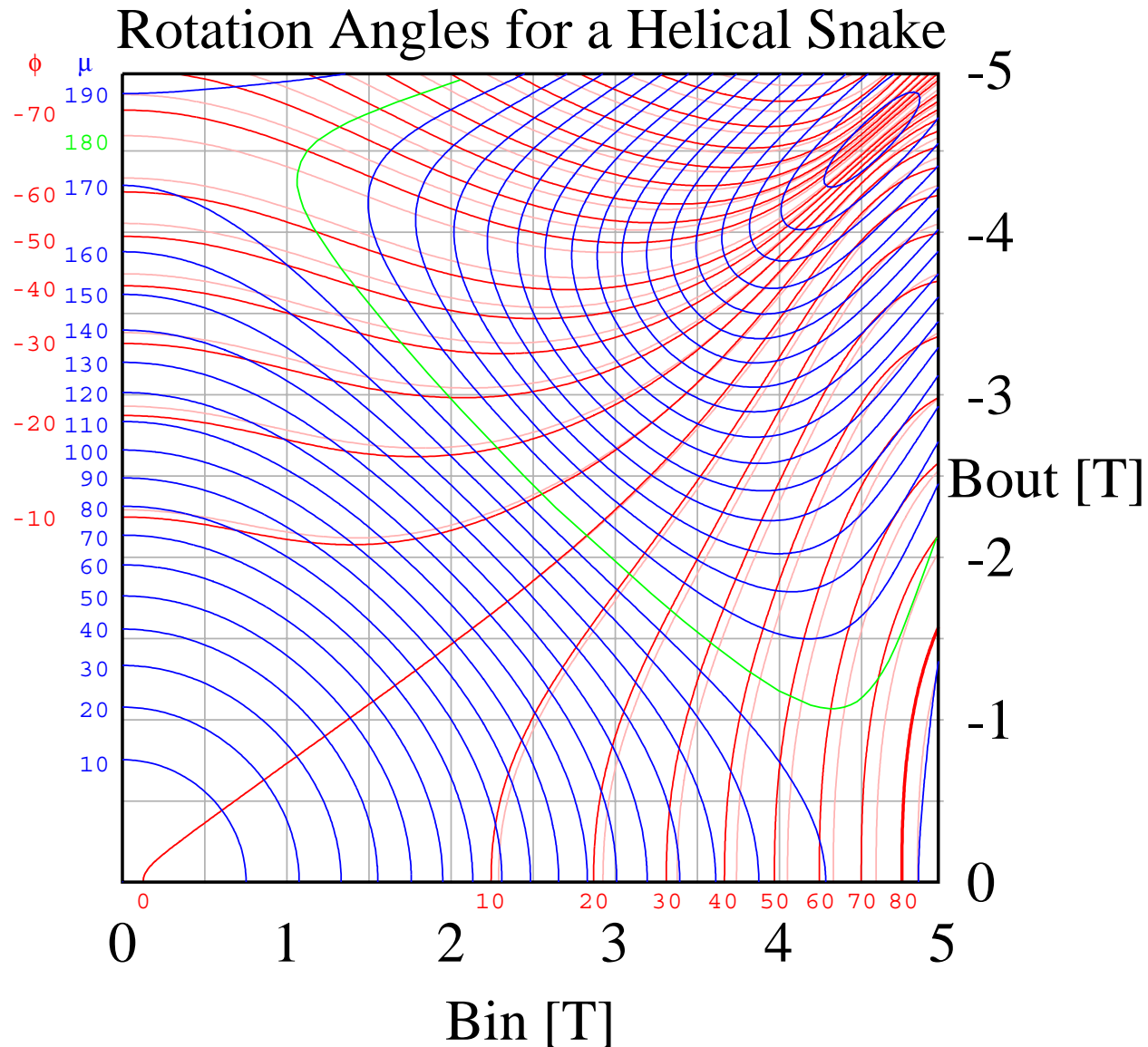
Trajectory and Spin through Snakes



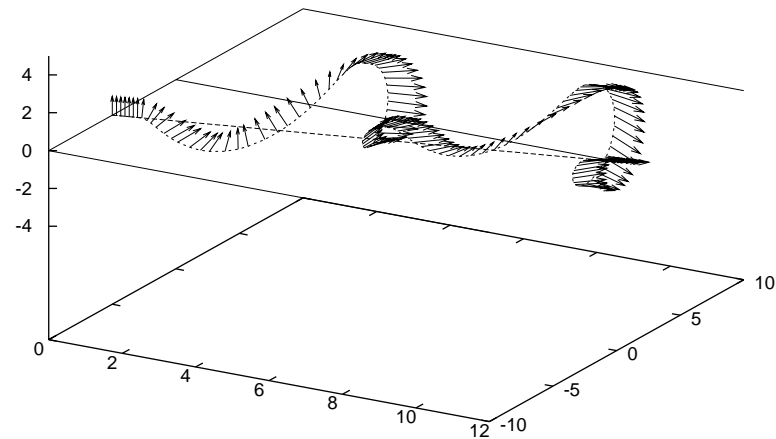
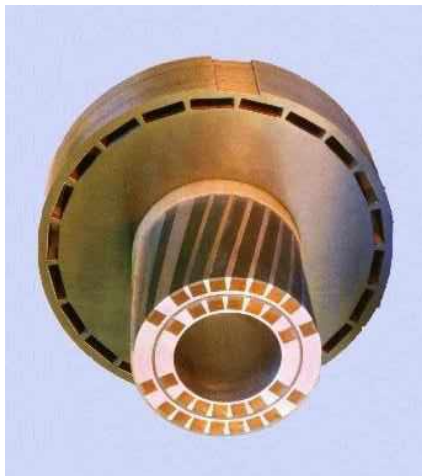
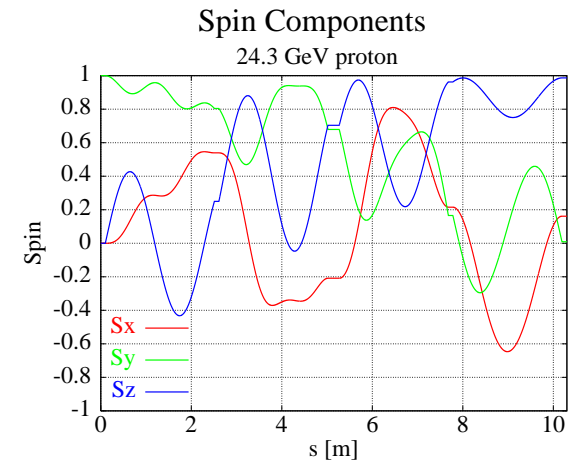
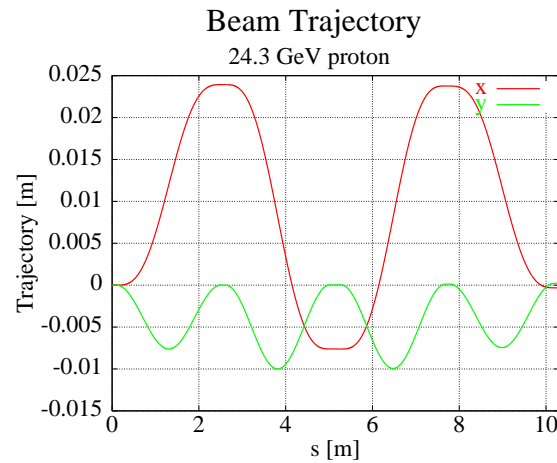
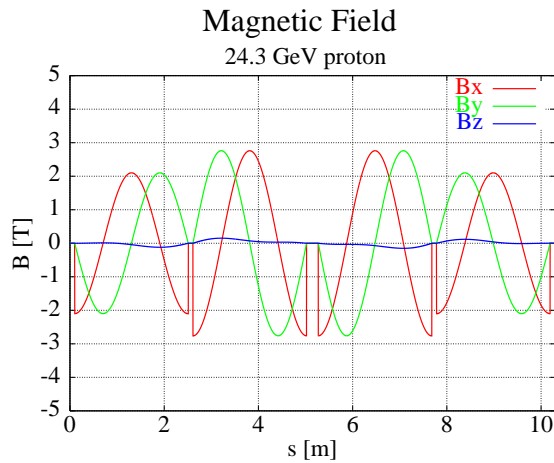


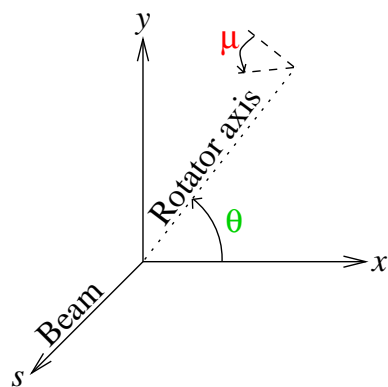
The rotation axis of the snake is ϕ , and μ is the rotation angle.

Note that the ϕ contours shift slightly from injection (red) at 25 GeV to storage (pink) at 250 GeV.

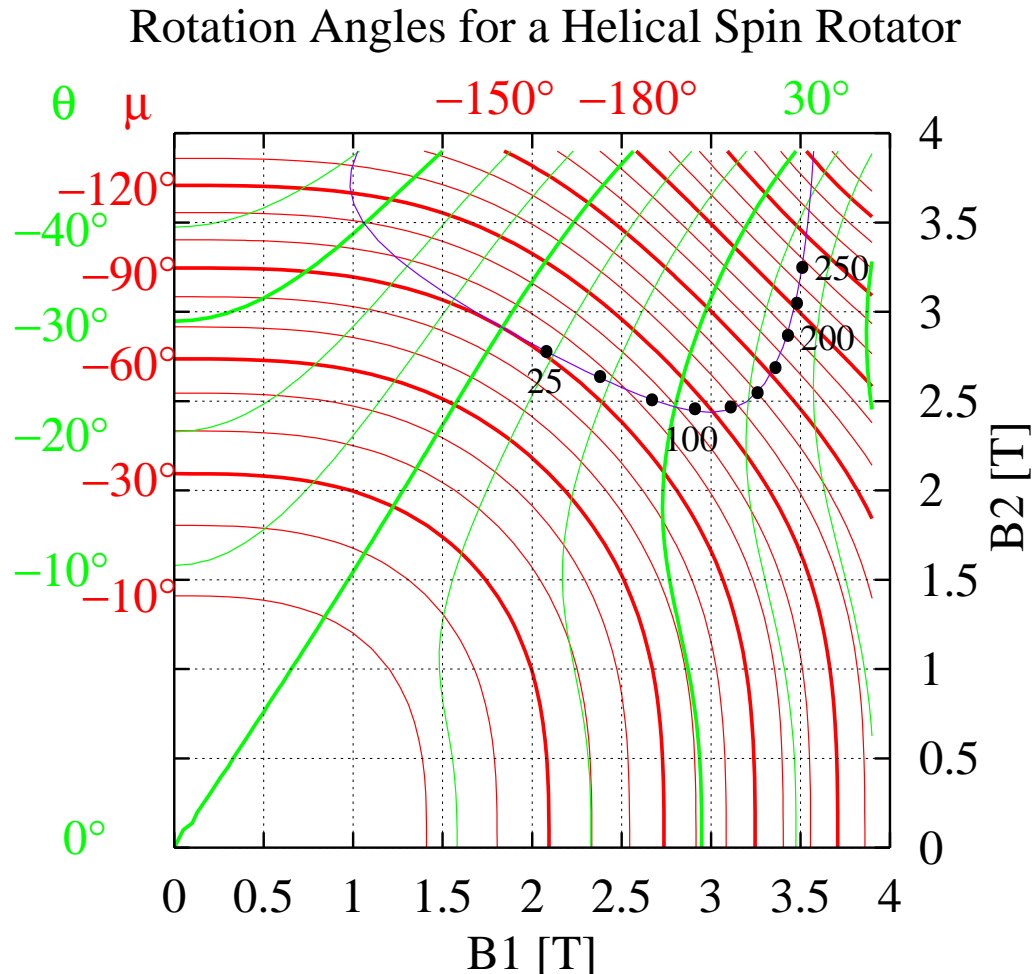


Helical Spin Rotators





The rotation axis of the spin rotator is in the x - y plane at an angle θ from the vertical. The spin is rotated by the angle μ around the rotation axis.



Note: Purple contour for rotation into horizontal plane.
Black dots show settings for RHIC energies in increments of 25 GeV from 25 to 250 GeV.

Rotator Axes and Precession

To precess the spin from vertical into the horizontal plane:

$$\sin \beta = \sin \mu \cos \theta$$

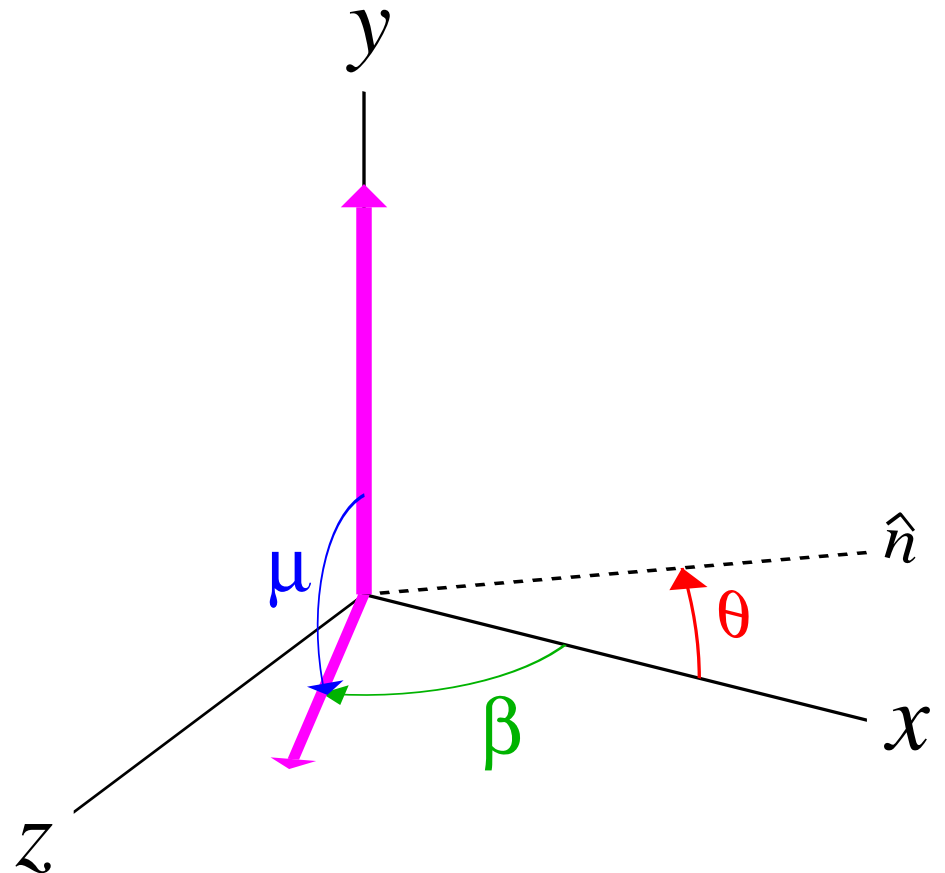
$$\cos \mu = -\tan^2 \theta$$

$$\mu \in [90^\circ, 270^\circ]$$

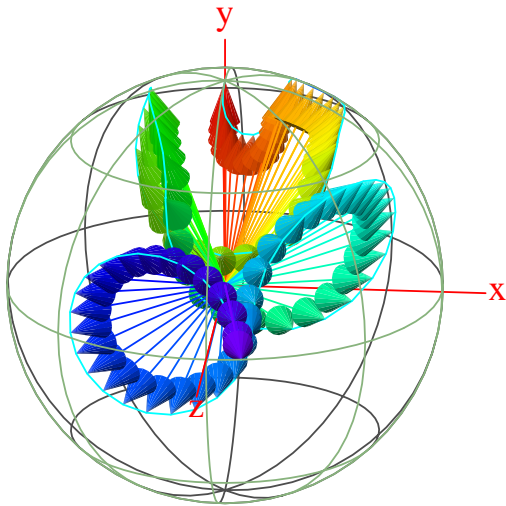
$$\theta \in [-45^\circ, 45^\circ] \cup [135^\circ, 225^\circ]$$

For longitudinal polarization want:

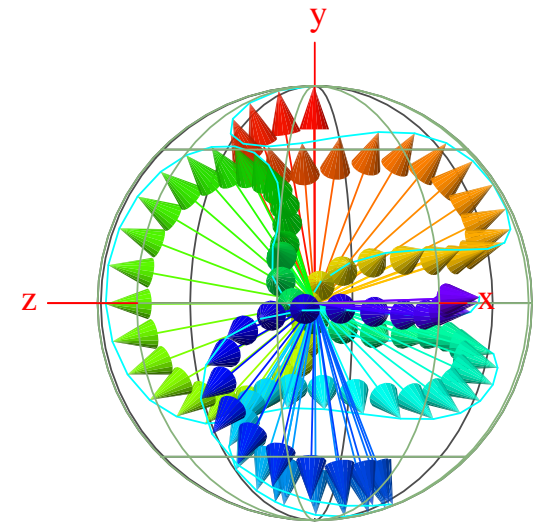
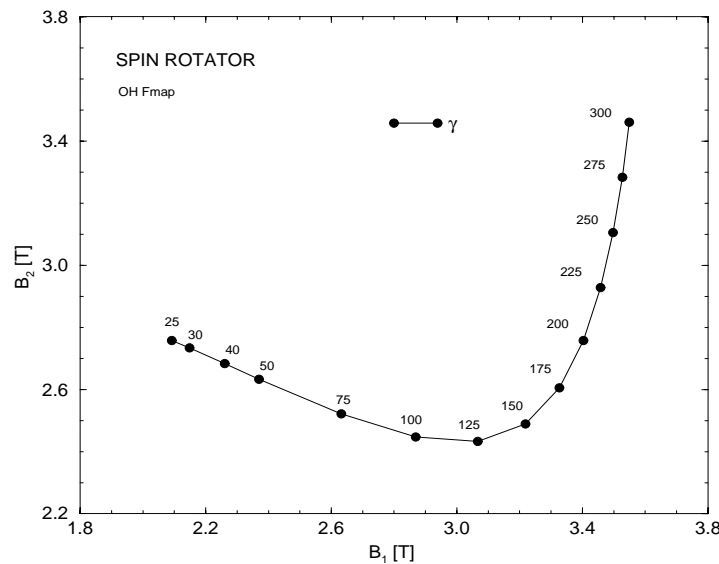
$$\beta = G\gamma \times \theta_{D0DX}$$



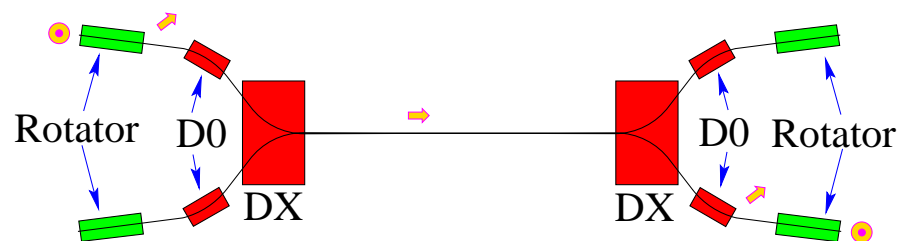
Compensation for D0-DX Bends



$E = 25 \text{ GeV}$
D0DX: 10° precession



$E = 250 \text{ GeV}$
D0DX: 100° precession



🎪 Formulae for a Single Rotator Helix 🎪

Parameters for a single RHIC rotator helix

Pitch: $k = \frac{2\pi}{\lambda}$, $\lambda = 2.41$ m $[+(-)]$ for right(left)-handed]

$$\kappa = \frac{q}{p}(1 + G\gamma)B$$

$$\text{Rotation axis: } \hat{n} = \frac{k\hat{z} + \kappa\hat{x}}{\sqrt{\kappa^2 + k^2}}$$

$$\text{Precession angle: } \alpha = 2\pi \left(\sqrt{1 + \left(\frac{\kappa}{k}\right)^2} - 1 \right)$$

$$\text{Transverse offset: } \Delta x = \frac{q}{p} \frac{B\ell}{k} = \frac{q}{p} \frac{\lambda^2}{2\pi} B$$

Scaling Snakes and Rotators to He³

Scaling of the field at maximum energy:

The maximum rigidity of the beams must be the same: $r_{\max} = \frac{p}{q} = 834 \text{ Tm}$

$$\gamma_{\text{He}^3} \simeq \frac{Z}{A} \gamma_p$$

Want the same precession, so κ must be the same.

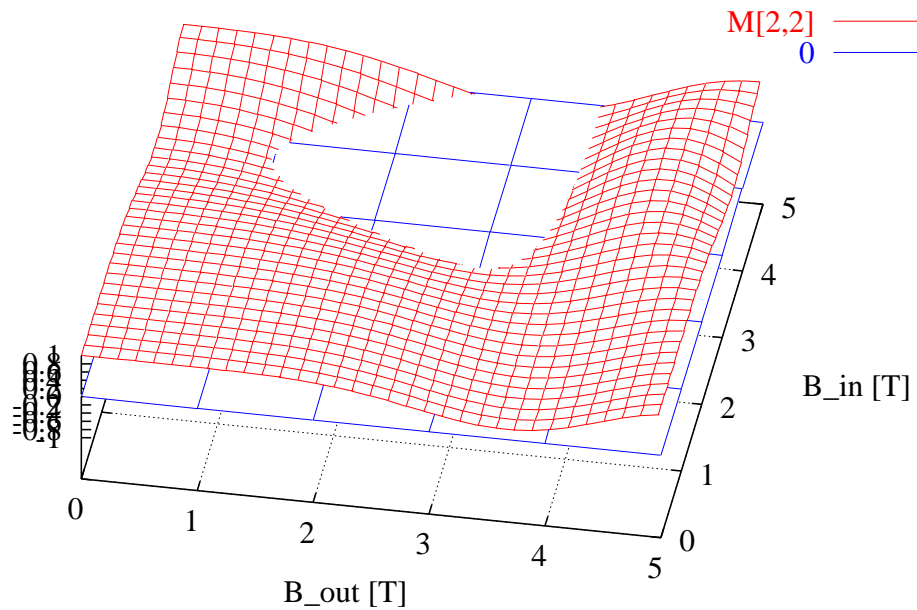
$$\begin{aligned} B_{\text{He}^3} &\simeq \frac{1 + G_p \gamma_p}{1 + G_{\text{He}^3} \gamma_{\text{He}^3}} B_p \\ &\simeq \frac{AG_p}{ZG_{\text{He}^3}} \simeq -0.643 \end{aligned}$$

Snake excursion at injection $r_{\text{inj}} = 81.1 \text{ Tm}$ (for protons):

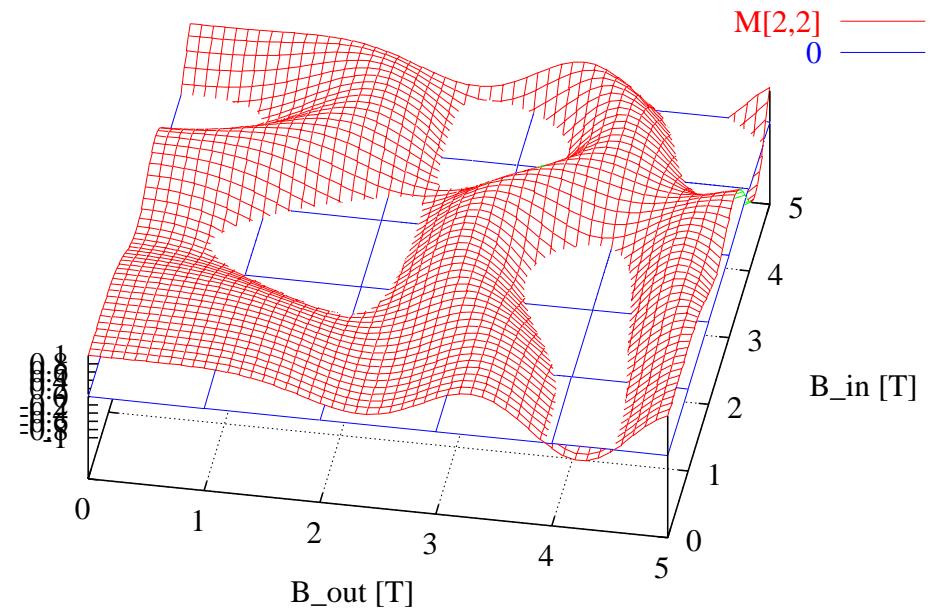
$$\Delta y = \begin{cases} 3.2 \text{ cm}, & \text{for protons} \\ -2.1 \text{ cm}, & \text{for He}^3 \end{cases}$$

Comparison of Rotators for He³ and p

Spin rotator contours for protons at 250 GeV



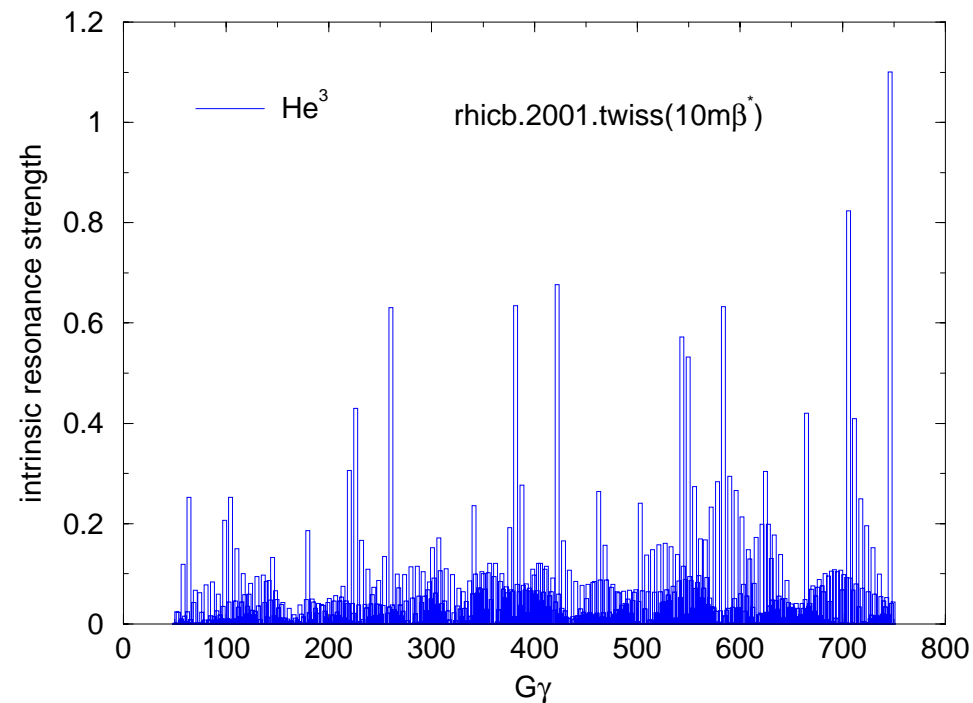
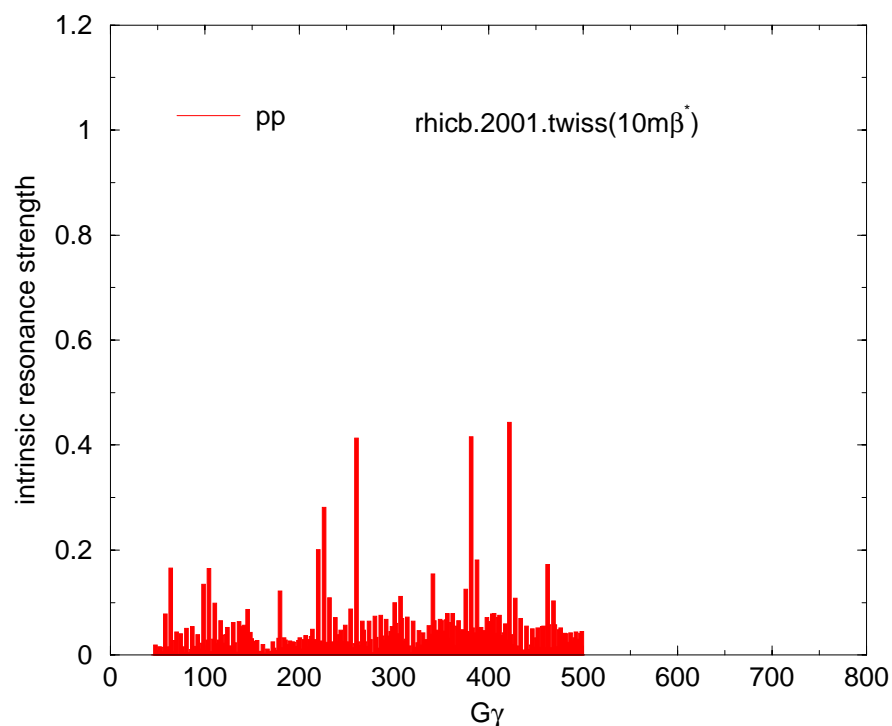
Spin rotator contour for He³ at 500 GeV



RHIC Spin Params for Diff. Species

| | p | $^2\text{H}^+$ | $^3\text{He}^{+2}$ | e^- |
|-----------------------------|------------|----------------|--------------------|--------------|
| m [GeV/c ²] | 0.9382720 | 1.8756127 | 2.8083912 | 0.0005109989 |
| $G = (g - 2)/2$ | 1.79284734 | -0.1426177 | -4.184 | 0.001159652 |
| mc^2/G [MeV] | 523.3418 | 13156.49 | 671.2216 | 440.6485 |
| $(p/q)_{\text{inj}}$ [Tm] | 81.113 | 81.113 | 81.027 | |
| U_{inj} [GeV] | 24.335 | 24.364 | 48.664 | |
| U_{inj}/n [GeV] | 24.335 | 12.182 | 16.221 | |
| γ_{inj} | 25.9362 | 13.0034 | 17.3280 | |
| $G\gamma_{\text{inj}}$ | 46.500 | -1.854 | -72.500 | |
| $(p/q)_{\text{store}}$ [Tm] | 833.904 | 833.904 | 833.904 | 33.356 |
| U_{store} [GeV] | 250.000 | 250.005 | 500.004 | 10 |
| U_{store}/n [GeV] | 250.000 | 125.003 | 166.668 | 10 |
| γ_{store} | 266.4473 | 133.2926 | 178.0394 | 19569.54 |
| $G\gamma_{\text{store}}$ | 477.699 | 19.062 | 744.917 | 22.6938 |

Depolarizing Resonances for Protons



$$|G\gamma|_{\max} = \begin{cases} 477, & \text{p} \\ 743, & \text{He}^3 \end{cases}$$

Summary

- ⌘ Spin precession and orbit excursions in snakes and rotators should work for protons eRHIC.
 - Snakes the same for protons.
 - If no “D0DX” bends for IR, then fields in rotators are essentially constant for all energies (like the snakes).
- ⌘ He³ requires less field in snakes and rotators.
 - Injection orbit excursions reduced.
- ⌘ $|G\gamma|_{\max}$ is higher for He³.
 - This needs to be investigated.

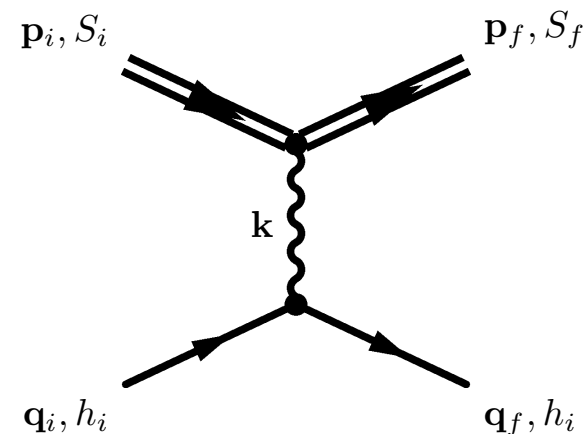
Depolarization from Electron Coolers

If we cool protons (or other light ions) with the electron cooler at injection, there can be depolarization due to the spin-flip interaction between the electron and proton beams.

- Need to calculate size of effect.

In principle, if the stable spin directions of the electron and proton beams were aligned, then we might expect to see a “spin-cooling” effect which would tend to polarize the protons to the same (or opposite) direction as the electron beam. Equilibrium polarization:

$$P_p = P_e \frac{\sigma_f^+ - \sigma_f^-}{\sigma_f^+ + \sigma_f^-}$$



Relevant literature

1. C. Bloch et al., *Spin-dependent scattering of polarized protons from a polarized ^3He internal gas target*, NIM **A 354**,437 (1995).
2. H. O. Meyer, *Effect of a polarized hydrogen target on the polarization of a stored proton beam*, Phys. Rev. **E 50**, 1485 (1994).
3. C. J. Horowitz and H. O. Meyer, *Polarizing Stored Beams by Interaction with Polarized Electrons*, Phys. Rev. Lett., **72**, 3981 (1994).
4. PAX Collaboration, *Letter of Intent for Antiproton-Proton Scattering Experiments with Polarization*, Jülich, Jan. 15, 2003.

Problems: Transversity + Helicity

- For the present design of electron coolers, the electrons are not polarized.
- If we had a polarized electron source, we could only consider \parallel electrons in the solenoid.
 - Spin precession about solenoid axis for \perp electrons

$$\frac{d\vec{S}}{ds} = \vec{S} \times \vec{\Omega}$$
$$\Omega = \frac{q}{p} \left[(1 + G\gamma)\vec{B}_{\perp} + (1 + G)\vec{B}_{\parallel} \right]$$

At injection $\gamma_{\text{inj}} = 25.9$, so for the electrons

$$\theta = \frac{q}{p}(1 + G)B_{\parallel}\ell \simeq \frac{1 + 0.00116}{0.0441[\text{Tm}]} \times 30[\text{Tm}] \sim 680 \text{ rad}$$

We could consider perhaps putting a proton rotator on each side of each cooler at the cost of

- $4 \times 1\text{M\$}$ for rotators
- $2 \times 11\text{m}$ of space for rotators leaving $< 10\text{m}$ for the cooler solenoid in a standard Q3–Q4 straight section.
- This would end up having the opposite effect for spin-up and spin-down proton bunches unless the cooler polarization was alternated in sync with the proton spin pattern.

§ eRHIC/ecooling aside: RHIC + H-Jet §

- Something which needs to be looked at for RHIC (no ecooling or eRHIC) with the Hydrogen Jet Polarimeter is the effect on the proton beam polarization from a polarized jet target (See earlier Refs.) Since we have a spin pattern of opposite polarizations for different bunches, we could expect to see a growing bias of the polarization levels of the opposite spins throughout a fill.
 - e.g.: spin-up polarization is larger than spin-down.
- Ordinary beam gas and electron clouds which should have no net polarization, may add to the general decay (equal for both signs) of polarization throughout a fill.
- For physics which has a factor of merit of $\mathcal{L}P^4$, small changes can make a big effect.

Summary

Need to estimate effect on proton (or other ion) polarization from:

- Decay of polarization from electron cooling.
- Decay of polarization from beam gas and electron clouds.
- Asymmetric up-down effect of polarized hydrogen jet.
- Symmetric effect of unpolarized hydrogen jet.